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## AET Mathematics Calculation Policy September 2014

## Summary

This calculation policy has been devised to support academies in understanding both the expectations for fluency of the 2014 curriculum and the progression of calculation concepts through a child's mathematical development.

## Principles

- This calculation policy is focused on developing proficiency with the expected formal written methods by the end of Year 6 and hence the progression guidance provided for each operation is designed to flow into the expected method as exemplified on the National Curriculum Appendix document (see page 6 for a summary of these).
- Specific practical equipment and approaches have been suggested for each age group to support children in developing the conceptual understanding that will enable them to move more rapidly and efficiently towards the formal written methods expected.
- It is recommended that teachers encourage children to simultaneously carry out the calculation practically using the equipment/representation suggested and to record this calculation step by step using the parallel formal written method.
- It is expected that academies will work towards the fluency goals for each age group but that, where necessary, teachers will use approaches and materials from earlier year groups to bridge any gaps in a child's understanding.
- Teachers should have an understanding of the expectations and progression for all year groups, regardless of which year group they teach.
- The 'Written Methods', 'With jottings ...or in your head' and 'Just know it' sections list the national curriculum expectations of the year group for calculation.
- The 'Developing Conceptual Understanding' section illustrates how to build children's understanding of the formal methods using a range of specific practical equipment and representations. The expected language for the formal methods is modelled in this section in the older year groups - this language should be used throughout whenever the formal method is used.
- The 'Foundations' section for each year group highlights the skills and knowledge that should be addressed on a regular basis within this year group to ensure that children have the requisite fluency to address the new approaches required.


## Addition

| Written <br> Methods | Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs | Add and subtract two two-digit numbers using concrete objects, pictorial representations progressing to formal written methods $\begin{array}{r} 46 \\ +27 \\ \hline 73 \\ \hline 1 \end{array}$ | Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction $\begin{array}{r} 423 \\ +\quad 88 \\ \hline \frac{511}{11} \end{array}$ | Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition where appropriate $\begin{array}{r} 2458 \\ +\quad 596 \\ \hline 3054 \\ \hline 111 \end{array}$ | Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) $\begin{array}{r} 23454 \\ +\quad 596 \\ \hline 24050 \\ \hline 111 \end{array}$ | Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Developing conceptual understanding | Use bonds of 10 to calculate bonds of 20 <br> 0000010000 <br> Count all <br> Count on <br> Count on, on number track, in 1 s | Number track / Number line - jumps of 1 then efficient jumps using number bonds $18+5=23$ <br> $46+27=73$ Count in tens then bridge. <br> $\begin{aligned} & \text { Partition and recombine } \\ & 46+27= \\ &= 0+13=73\end{aligned}$ <br> $24+10$ <br> $+10$ <br> $+10=54$ |  | Place Value Counters $2458+596$ <br> Show 2458 and 596 <br> Combine the 1s. Exchange ten 1s for a 10 counter. <br> Combine the 10s. Exchange ten 10 s for a 100 counter. <br> Combine the 100s. Exchange ten 100s for a 1000 counter <br> Read final answer Three thousand and fifty-four. |  |  |
| With jottings <br> ... or in your head | Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7=\square-9$ | Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: <br> * a two-digit number and ones <br> * a two-digit number and tens <br> * two two-digit numbers <br> * adding three one-digit numbers | Add and subtract numbers mentally, including: <br> * a three-digit number and ones <br> * a three-digit number and tens <br> * a three-digit number and hundreds | Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why | Add and subtract numbers mentally with increasingly large numbers | Perform mental calculations, including with mixed operations and large numbers |
| Just know it! | Represent \& use number bonds and related subtraction facts within 20 Add and subtract one-digit and twodigit numbers to 20 , including zero | Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 |  |  |  |  |
| Year | 1 | 2 | 3 | 4 | 5 | 6 |
| Foundations | 1 more | 10 more Number bonds: 20, 12, 13 | Add multiples of 10, 100 | Add multiples of 10s, 100s, 1000s | Add multiples of $10 \mathrm{~s}, 100 \mathrm{~s}$, 1000s, tenths, | Add multiples of $10 \mathrm{~s}, 100 \mathrm{~s}, 1000 \mathrm{~s}$, tenths, hundredths |
|  | Number bonds: 5, 6 | Number bonds: 14,15 <br> Add 1 digit to 2 digit by bridging. | Add single digit bridging through boundaries | Fluency of 2 digit +2 digit | Fluency of 2 digit + 2 digit including with decimals | Fluency of 2 digit +2 digit including with decimals |
|  | Largest number first. Number bonds: 7,8 | Partition second number, add tens then ones | Partition second number to add Pairs of 100 | Partition second number to add Decimal pairs of 10 and 1 | Partition second number to add | Partition second number to add |
|  | Add 10. Number bonds: 9,10 | Add 10 and multiples. Number bonds: 16 and 17 | Use near doubles to add | Use near doubles to add | Use number facts, bridging and place value | Use number facts, bridging and place value |
|  | Ten plus ones. Doubles up to 10 | Doubles up to 20 and multiples of 5 Add near multiples of 10 . | Add near multiples of 10 and 100 by rounding and adjusting | Adjust both numbers before adding Add near multiples | Adjust numbers to add | Adjust numbers to add |
|  | Use number bonds of 10 to derive bonds of 11 | Number bonds: 18, 19 Partition and recombine | Parrition and recombine | Parrition and recombine | Parrition and recombine | Partition and recombine |

Subtraction


## Multiplication



Division

| Written <br> Methods |  | Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication（ $\times$ ），division（ $\div$ ）and equals（＝）signs | Write and calculate mathematical statements for $\div$ using the $x$ tables they know progressing to formal written methods． |  | Divide numbers up to <br> 4 digits by a one－digit $194 \div 6$ number using the formal written method of short $3 \quad 2$ $6 \longdiv { 1 9 ^ { 1 } 2 }$ division and interpret remainders $\quad 192 \div 6$ appropriately for the $=32$ context | Divide numbers up to 4－digits by a two－digit whole number using the formal written method of short division where appropriate for the context $\begin{aligned} & 564 \div 13 \\ & 1 3 \longdiv { 5 6 ^ { 4 } 4 } 5 \end{aligned}$ <br> Known multiplication facts： <br> $13,26,39,52,65, \ldots$ <br> $10 \times 13=130,20 \times 13=260 \ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Developing conceptual understanding | $6 \div 2=3$ by sharing into 2 groups and by grabbing groups of 2 <br> How many 2s？ | $15 \div 3=5$ in each group（sharing） <br> Link to fractions <br> $15 \div 3=5$ groups of 3 （grouping） <br> － <br> $10 \div 2=5$ <br> Use language of division linked to tables $\square$ <br> How many 2s？ | Grouping using partitioning $43 \div 3$ If I know $10 \times 3 \ldots$ <br> Use language of division linked to tables $\square$ <br> How many 3s？ | Grouping using partitioning $196 \div 6$ If I know $3 \times 6 \ldots$ then $30 \times 6 \ldots$ <br> ＇Chunking up＇on a number line $196 \div 6=32$ r 4 <br> Use language of division linked to tables． $\square$ | $192 \div 6$ using place value counters to support written method <br> 3 groups so that is $30 \times 6$ ， exchange remaining 10 for ten 1 s <br> So $192 \div 6=32$ | $564 \div 13=43 \text { r } 5=43 \frac{5}{13}$ <br> Divide numbers up to 4 digits by a two－digit whole number using the formal written method of long division，and interpret remainders as whole number remainders，fractions，or by rounding，as appropriate for the context |
| With jottings <br> ．．．or in your head ．．．． | Solve one－step problems involving multiplication and division，by calculating the answer using concrete objects，pictorial representations and arrays with the support of the teacher | Show that multiplication of two numbers can be done in any order （commutative）and division of one number by another cannot Solve problems involving multiplication and division，using materials，arrays，repeated addition， mental methods，and multiplication and division facts，including problems in contexts | Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know，including for two－digit numbers times one－digit numbers， using mental methods | Use place value，known and derived facts to multiply and divide mentally， including：multiplying by 0 and 1 ； dividing by 1 ；multiplying together three numbers Recognise and use factor pairs and commutativity in mental calculations | Multiply and divide numbers mentally drawing upon known facts Multiply and divide whole numbers and those involving decimals by 10 ， 100 and 1000 | Perform mental calculations，including with mixed operations and large numbers |
| Just know it！ | Count in multiples of twos，fives and tens | Recall and use x and $\div$ facts for the 2 ， 5 and $10 \times$ tables，including recognising odd and even numbers． | Recall and use x and $\div$ facts for the 3 ， 4 and 8 times tables | Recall x and $\div$ facts for x tables up to $12 \times 12$ ． | Recall prime numbers up to 19 know and use the vocabulary of prime numbers，prime factors and composite （non－prime）numbers |  |
| Year | 1 | 2 | 3 | 4 | 5 | 6 |
| Foundations | Count back in 2 s | Division facts（ $2 \times$ table） | Review division facts（ $2 \mathrm{x}, 5 \mathrm{x}, 10 \mathrm{x}$ table） | Division facts（ $4 \mathrm{x}, 8 \mathrm{x}$ tables） 10 times smaller | Division facts（ $4 \mathrm{x}, 8 \mathrm{x}$ tables） 100， 1000 times smaller | Division facts（up to $12 \times 12$ ） |
|  | Count back in 10s | Division facts（ $10 \times$ table） | Division facts（ $4 \times$ table） | Division facts（ $3 \mathrm{x}, 6 \mathrm{x}, 12 \mathrm{x}$ tables） | Division facts（ $3 x, 6 x, 12 x$ tables） Partition to divide mentally | Partition to divide mentally |
|  | Halves up to 10 | Halves up to 20 | Halve two digit numbers | Halve larger numbers and decimals | Halve larger numbers and decimals | Halve larger numbers and decimals |
|  | Count back in 5 s | Division facts（ $5 \times$ table） | Division facts（ $8 \times \mathrm{table}$ ） | Division facts（3x，9x tables） | Division facts（ $3 \mathrm{x}, 9 \mathrm{x}$ tables） 100， 1000 times smaller | Division facts（up to $12 \times 12$ ） |
|  | Halve multiples of 10 | Count back in 3 s | Division facts（ $3 \times$ table） | Division facts（11x，7x tables） | Review division facts（11x，7x tables） Partition decimals to divide mentally | Partition to divide mentally |
|  | How many 2s？5s？10s？ | Review division facts（ $2 x, 5 x, 10 x$ table） | Division facts（ $6 \times$ table）or review others | Division facts（ $6 \mathrm{x}, 12 \mathrm{x}$ tables） | Review division facts（ $6 x, 12 x$ tables） Halve larger numbers and decimals | Halve larger numbers and decimals |

Expectations of Calculation in Year 6


## Glossary of Terms

2-digit number- a number with 2 digits like $23,45,12$ or 60
3-digit number - a number with 3 digits like 123, 542, 903 or 561

Addition facts - knowing that $1+1=2$ and $1+3=4$ and $2+5=7$. Normally we only talk about number facts with totals of 20 and under.
Array - An array is an arrangement of a set of numbers or objects in rows and columns -it is mostly used to show how you can group objects for repeated addition or subtraction.

Bead String/Bar - a string with (usually 100) beads on, grouped by colour in tens. The bead string is a good bridge between a number track and a number line as it maintains the cardinality of the numbers whilst beginning to develop the concepts of counting 'spaces' rather than objects.

Bridging - when a calculation causes you to cross a 'ten boundary' or a 'hundred boundary' e.g. $85+18$ will bridge 100.

Compact vertical - the name of the recommended written method for addition whereby the numbers are added in columns, 1 s first then 10 s and so on. Where the total exceeds 10 , the ten 1 s are exchanged for a 10 and written below the answer line. Sometimes referred to as 'carrying'.

Concrete apparatus - objects to help children count and calculate- these are most often cubes (multilink) but can be anything they can hold and move including Cuisenaire rods, Dienes rods (hundreds, tens and units blocks), straws, Numicon, Place Value counters and much more.

Count all - when you add by counting all the items/objects e.g. to add 11 and 5 you would count out 11, then count out 5 , then put them together and count them all to get 16 .

Count on - when you add (or sometimes subtract) by counting onwards from a given number. E.g. to add 11 and 5 you would count on 5 from 11 i.e. 12,13 , 14, 15, 16

Decimal number - a number with a decimal point e.g. 2.34 (said as two point three four)

Decomposition - the name of the recommended written method for subtraction whereby the smaller number is subtracted from the larger, 1 s first then 10 s and so on. Where the subtraction cannot be completed as the second number is larger than the first, a 10 is exchanged for ten 1 s to facilitate this. This is the traditional 'borrowing' form of column method, which is different to the 'payback' method.

Dienes Rods (or Base 10) - this is a set of practical equipment that represents the numbers to help children with place value and calculation. The Dienes rods show $1 \mathrm{~s}, 10 \mathrm{~s}, 100 \mathrm{~s}$ and 1000 s as blocks of cubes that children can then combine. Dienes rods do not break up so the child has to 'exchange' them for smaller or larger blocks where necessary.

Difference - the gap between numbers that is found by subtraction e.g. 7-5 can be read as ' 7 take away 5' or as the 'difference between 7 and 5 '

Dividend - the number being divided in a calculation

Divisor - the smaller number in a division calculation.

Double - multiply a number by 2

Efficient Methods - the method(s) that will solve the calculation most rapidly and easily

Equals - is worth the same as (be careful not to emphasise the use of = to show the answer)

Exchanging - Swapping a '10' for ten '1s' or a '100' for ten '10s' or vice versa (used in addition and subtraction when 'moving' 'ten' or a 'hundred' from its column into the next column and splitting it up). Heavily relied upon for addition and subtraction of larger numbers. Skills in this can be built up practically with objects, then Dienes rods/base 10, then place value counters before relying on a solely written method.

Expanded Multiplication - a method for multiplication where each stage is written down and then added up at the end in a column

Factor - a number that divides exactly into another number, without remainder

Grid method - a method for multiplying two numbers together involving partitioning and multiplying each piece separately.

Grouping - an approach to division where the dividend is split into groups of the size of the divisor and the number of groups created are then counted.

Half - a number, shape or quantity divided into 2 equal parts

Halve - divide a number by 2

Integer - a whole number (i.e. one with no decimal point)

Inverse - the opposite operation. For example, addition is the inverse of subtraction and multiplication is the inverse of division.

Known Multiplication Facts - times tables and other number facts that can be recalled quickly to support with larger or related calculations e.g. if you know $4 \times 7$ then you also know $40 \times 70,4 \times 0.7$ etc.

Long Division - formal written of division where the remainders are calculated in writing each time (extended version of short division)

Long Multiplication - formal written method of column multiplication

Multiple - a number which is an exact product of another number i.e. a number which is in the times table of another number

Number bonds - 2 numbers that add together to make a given total, e.g. 8 and 2 bond to 10 or 73 and 27 bond to 100

Number line - a line either with numbers or without (a blank numberline).
The number line emphasises the continuous nature of numbers and the existence of 'in-between' numbers that are not whole. It is based around the gaps between numbers.

Children use this tool to help them count on or count back for addition of subtraction. As they get older, children will count in 'jumps' on a number line e.g to add 142 to a number they may 'jump' 100 and then 40 and then 2 . The number line is sometimes used in multiplication and division but can be time consuming.

Number track - a sequence of numbers, each inside its own square. It is a simplified version of the number line that emphasises the whole numbers.

Numicon - practical maths equipment that teaches children the names and values of numbers 1-10 initially but them helps them with early addition, subtraction, multiplication and division. Numicon is useful for showing the real value of a number practically.

One-Step Calculation - a calculation involving only one operation e.g. addition. Usually the child must decide what that operation is.

Partition - split up a larger number into parts, such as the hundreds, tens and units e.g. 342 can be partitioned into 300 and 40 and 2

Place Value - the value of a digit created by its position in a number e.g. 3 represents thirty in 234 but three thousand in 3567

Recombine - for addition, once you have partitioned numbers into hundreds, tens and units then you have to add then hundreds together, then add the tens to that total, then add the units to that total

Remainder - a whole number left over after a division calculation

Repeated addition - repeatedly adding groups of the same size for multiplication

Scaling - an approach to multiplication whereby the number is 'scaled up' by a factor of the multiplier e.g. $4 \times 3$ means 4 scaled up by a factor of 3 .
Sharing - an approach to division whereby the dividend is shared out into a given number of groups (like dealing cards)
Short Division - traditional method for division with a single digit divisor (this is a compact version of long division, sometimes called 'bus stop')
Significant digit - the digit in a number with the largest value e.g. in 34 the most significant digit is the 3 , as it has a value of ' 30 ' and the ' 4 ' only has a value of ' 4 '

Single digit - a number with only one digit. These are always less than 10.
Sum - the total of two or more numbers (it implies addition). Sum should not be used as a synonym for calculation.
Two-step calculation - a calculation where two different operations must be applied e.g. to find change in a shop you will usually have to add the individual prices and then subtract from the total amount. Usually the child has to decide what these two operations are and the order in which they should be applied.

